The Entire 1988 AP Calculus AB Examination and Key

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Advanced Placement

Calculus AB Examination

1988

CONTENTS

INTRODUCTION

All Advanced Placement Examinations, except portfolio evaluations in Studio Art, have both multiple-choice and free-response sections. The complete 1988 AP Calculus AB Examination is reproduced in this booklet. (Copies of the examination are also available in sets of 10 for classroom use, $20 per set; see ordering information on page 6.) The number of questions and the time allowed for the administration of each section of the examination are:

Multiple-choice, Section I:	45 questions 1 hour and 30 minutes
Free-response, Section II:	6 problems 1 hour and 30 minutes

The free-response and multiple-choice sections are designed to complement each other and, as a complete examination, to meet the overall course objectives and examination specifications given in the *AP Course Description: Mathematics (Calculus AB, Calculus BC)*.

The multiple-choice questions (Section I) cover a wide range of content within the examination specifications and vary in difficulty and complexity. Thus, the breadth of a student's knowledge and skills can be measured by the complete array of multiple-choice questions, with no question bearing undue weight in the total Section I score. The free-response questions (Section II) are designed to measure a student's ability to recall and apply calculus and other mathematical principles in a more extensive way at greater depth. The multiple-choice and the free-response questions are analyzed individually and collectively after each administration, and the conclusions are used in the setting of the grades as well as in improving future examinations.

In addition to a copy of the 1988 AP Calculus AB Examination, this booklet contains data concerning the performance of AP candidates on the examination, solutions and grading standards for the free-response questions, and specific information about the scoring of both sections of the examination. The 1988 examination is comparable in content and difficulty to other Calculus AB examinations in recent years. Because it is not possible to predict accurately the performance on each question, there is some variation in the difficulty of the examinations from year to year. However, this variation is considered in determining the reported grades in order to avoid penalizing candidates who may have taken the examination in a year in which a more difficult version was given.

It should be noted that although Calculus AB and Calculus BC are different courses, they cover some of the same topics. However, since 1986 the two examinations have not contained any questions common to both examinations. The entire 1988 AP Calculus BC Examination with a key and data is available in a separate booklet.

Additional multiple-choice questions from previous AP Examinations are in *AP Course Description: Mathematics* ($5). The free-response portion (Section ı of the examination is available annually, and a complete set of the free-respon questions for the past five years also may be ordered ($2). To order any of the above materials (prepaid), write to AP Program, P.O. Box 6670, Princeton, NJ 08541-6670.

1988

MATHEMATICS: CALCULUS AB

CALCULATORS AND REFERENCE MATERIALS MAY NOT BE USED IN THE EXAMINATION ROOM DURING THE TESTING PERIOD.

Three hours are allotted for this examination: 1 hour and 30 minutes for Section I, which consists of multiple-choice questions; and 1 hour and 30 minutes for Section II, which consists of longer problems. In determining your grade, the two sections are given equal weight. Section I is printed in this examination booklet; Section II, in a separate booklet.

SECTION I

Time—1 hour and 30 minutes

Number of questions—45

Percent of total grade—50

This examination contains 45 multiple-choice questions. Therefore, please be careful to fill in only the ovals that are preceded by numbers 1 through 45 on your answer sheet.

General Instructions

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE INSTRUCTED TO DO SO.

INDICATE ALL YOUR ANSWERS TO QUESTIONS IN SECTION I ON THE SEPARATE ANSWER SHEET ENCLOSED. No credit will be given for anything written in this examination booklet, but you may use the booklet for notes or scratchwork. After you have decided which of the suggested answers is best, COMPLETELY fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely.

Example:

What is the arithmetic mean of the numbers 1, 3, and 6?

(A) 1 (B) $\frac{7}{3}$ (C) 3

(D) $\frac{10}{3}$ (E) $\frac{7}{2}$

Sample Answer

Ⓐ Ⓑ Ⓒ ● Ⓔ

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. In this section of the examination, as a correction for haphazard guessing, one-fourth of the number of questions you answer incorrectly will be subtracted from the number of questions you answer correctly. It is improbable, therefore, that mere guessing will improve your score significantly; it may even lower your score, and it does take time. If, however, you are not sure of the best answer but have some knowledge of the question and are able to eliminate one or more of the answer choices as wrong, your chance of answering correctly is improved, and it may be to your advantage to answer such a question.

Use your time effectively, working as rapidly as you can without losing accuracy. Do not spend too much time on questions that are too difficult. Go on to other questions and come back to the difficult ones later if you have time. It is not expected that everyone will be able to answer all the multiple-choice questions.

CALCULUS AB

SECTION I

Time—1 hour and 30 minutes

Number of questions—45

Percent of total grade—50

Directions: Solve each of the following problems, using the available space for scratchwork. Then decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in this examination booklet. Do not spend too much time on any one problem.

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $y = x^2 e^x$, then $\frac{dy}{dx} =$

(A) $2xe^x$

(B) $x(x + 2e^x)$

(C) $xe^x(x + 2)$

(D) $2x + e^x$

(E) $2x + e$

GO ON TO THE NEXT PAGE

2. What is the domain of the function f given by $f(x) = \frac{\sqrt{x^2 - 4}}{x - 3}$?

(A) $\{x:\ x \neq 3\}$

(B) $\{x:\ |x| \leqq 2\}$

(C) $\{x:\ |x| \geqq 2\}$

(D) $\{x:\ |x| \geqq 2 \text{ and } x \neq 3\}$

(E) $\{x:\ x \geqq 2 \text{ and } x \neq 3\}$

---

3. A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

(A) $e^2 - 1$

(B) $e - 1$

(C) $2e$

(D) e^2

(E) $\frac{e^3}{3}$

GO ON TO THE NEXT PAGE

4. The graph of $y = \dfrac{-5}{x-2}$ is concave downward for all values of x such that

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x >$

5. $\int \sec^2 x \, dx =$

(A) $\tan x + C$

(B) $\csc^2 x + C$

(C) $\cos^2 x + C$

(D) $\dfrac{\sec^3 x}{3} + C$

(E) $2 \sec^2 x \tan x + C$

GO ON TO THE NEXT PAGE

6. If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) $\frac{\ln x - 1}{x^2}$ (D) $\frac{1 - \ln x}{x^2}$ (E) $\frac{1 + \ln x}{x^2}$

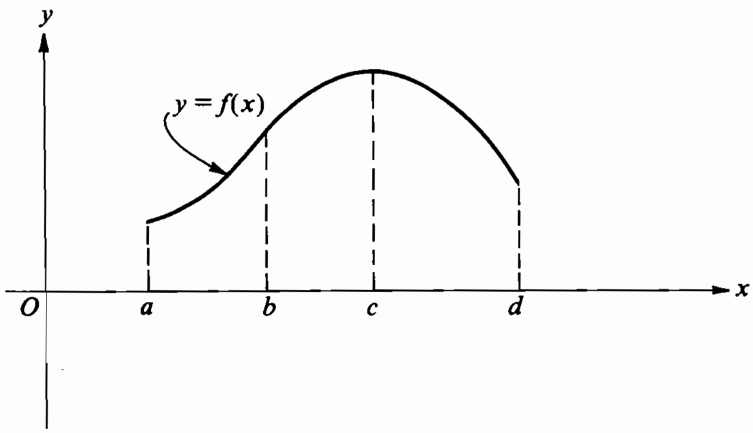
---

7. $\int \frac{x\,dx}{\sqrt{3x^2 + 5}} =$

(A) $\frac{1}{9}(3x^2 + 5)^{\frac{3}{2}} + C$ (B) $\frac{1}{4}(3x^2 + 5)^{\frac{3}{2}} + C$ (C) $\frac{1}{12}(3x^2 + 5)^{\frac{1}{2}} + C$

(D) $\frac{1}{3}(3x^2 + 5)^{\frac{1}{2}} + C$ (E) $\frac{3}{2}(3x^2 + 5)^{\frac{1}{2}} + C$

GO ON TO THE NEXT PAGE



8. The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

GO ON TO THE NEXT PAGE

9. If $x + 2xy - y^2 = 2$, then at the point $(1, 1)$, $\frac{dy}{dx}$ is

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{3}{2}$ (E) nonexistent

---

10. If $\int_0^k (2kx - x^2)dx = 18$, then $k =$

(A) -9 (B) -3 (C) 3 (D) 9 (E) 18

GO ON TO THE NEXT PAGE

11. An equation of the line tangent to the graph of $f(x) = x(1 - 2x)^3$ at the point $(1, -1)$ is

(A) $y = -7x + 6$

(B) $y = -6x + 5$

(C) $y = -2x$ ·

(D) $y = 2x - 3$

(E) $y = 7x - 8$

---

12. If $f(x) = \sin x$, then $f'\left(\frac{\pi}{3}\right) =$

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{2}}{2}$

(D) $\frac{\sqrt{3}}{2}$

(E) $\sqrt{3}$

GO ON TO THE NEXT PAGE

13. If the function f has a continuous derivative on [0, c], then $\int_0^c f'(x)\,dx =$

(A) $f(c) - f(0)$ (B) $|f(c) - f(0)|$ (C) $f(c)$ (D) $f(x) + c$ (E) $f''(c) - f''(0)$

---

14. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta =$

(A) $-2(\sqrt{2}-1)$ (B) $-2\sqrt{2}$ (C) $2\sqrt{2}$ (D) $2(\sqrt{2}-1)$ (E) $2(\sqrt{2}+1)$

GO ON TO THE NEXT PAGE

15. If $f(x) = \sqrt{2x}$, then $f'(2) =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) 1 (E) $\surd$

---

16. A particle moves along the x-axis so that at any time $t \geqq 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$. For what values of t is the particle at rest?

(A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

GO ON TO THE NEXT PAGE

17. $\int_0^1 (3x - 2)^2 \, dx =$

(A) $-\frac{7}{3}$ (B) $-\frac{7}{9}$ (C) $\frac{1}{9}$ (D) 1 (E) 3

---

18. If $y = 2\cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2} =$

(A) $-8\cos\left(\frac{x}{2}\right)$ (B) $-2\cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$ (D) $-\cos\left(\frac{x}{2}\right)$ (E) $-\frac{1}{2}\cos\left(\frac{x}{2}\right)$

GO ON TO THE NEXT PAGE

19. $\int_2^3 \frac{x}{x^2+1}\,dx =$

(A) $\frac{1}{2}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln 2$ (C) $\ln 2$ (D) $2\ln 2$ (E) $\frac{1}{2}\ln$

---

20. Let f be a polynomial function with degree greater than 2. If $a \neq b$ and $f(a) = f(b) = 1$, which of the following must be true for at least one value of x between a and b?

I. $f(x) = 0$
II. $f'(x) = 0$
III. $f''(x) = 0$

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

GO ON TO THE NEXT PAGE

21. The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) 2 (E) $\frac{14}{3}$

---

22. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

GO ON TO THE NEXT PAGE

23. If $f'(x) = \cos x$ and $g'(x) = 1$ for all x, and if $f(0) = g(0) = 0$, then $\lim_{x \to 0} \frac{f(x)}{g(x)}$ is

(A) $\frac{\pi}{2}$ (B) 1 (C) 0 (D) -1 (E) nonexistent

---

24. $\frac{d}{dx}(x^{\ln x}) =$

(A) $x^{\ln x}$ (B) $(\ln x)^x$ (C) $\frac{2}{x}(\ln x)(x^{\ln x})$ (D) $(\ln x)(x^{\ln x - 1})$ (E) $2(\ln x)(x^{\ln x})$

GO ON TO THE NEXT PAGE

25. For all $x > 1$, if $f(x) = \int_1^x \frac{1}{t}\,dt$, then $f'(x) =$

(A) 1 (B) $\frac{1}{x}$ (C) $\ln x - 1$ (D) $\ln x$ (E) e^x

---

26. $\int_0^{\frac{\pi}{2}} x \cos x \, dx =$

(A) $-\frac{\pi}{2}$ (B) -1 (C) $1 - \frac{\pi}{2}$ (D) 1 (E) $\frac{\pi}{2} - 1$

GO ON TO THE NEXT PAGE

27. At $x = 3$, the function given by $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geqq 3 \end{cases}$ is

(A) undefined

(B) continuous but not differentiable

(C) differentiable but not continuous

(D) neither continuous nor differentiable

(E) both continuous and differentiable

---

28. $\int_1^4 |x - 3|\, dx =$

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$ (E) 5

GO ON TO THE NEXT PAGE

29. The $\lim\limits_{h \to 0} \dfrac{\tan 3(x + h) - \tan(3x)}{h}$ is

(A) 0 (B) $3 \sec^2 (3x)$ (C) $\sec^2 (3x)$ (D) $3 \cot (3x)$ (E) nonexistent

---

30. A region in the first quadrant is enclosed by the graphs of $y = e^{2x}$, $x = 1$, and the coordinate axes. If the region is rotated about the y-axis, the volume of the solid that is generated is represented by which of the following integrals?

(A) $2\pi \int_0^1 xe^{2x}\, dx$

(B) $2\pi \int_0^1 e^{2x}\, dx$

(C) $\pi \int_0^1 e^{4x}\, dx$

(D) $\pi \int_0^e y \ln y\, dy$

(E) $\frac{\pi}{4} \int_0^e \ln^2 y\, dy$

GO ON TO THE NEXT PAGE

31. If $f(x) = \frac{x}{x+1}$, then the inverse function, f^{-1}, is given by $f^{-1}(x) =$

(A) $\frac{x-1}{x}$ (B) $\frac{x+1}{x}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x+1}$ (E) x

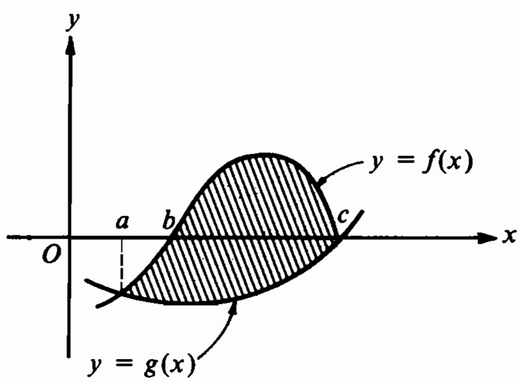
32. Which of the following does NOT have a period of π ?

(A) $f(x) = \sin\left(\frac{1}{2}x\right)$
(B) $f(x) = |\sin x|$
(C) $f(x) = \sin^2 x$
(D) $f(x) = \tan x$
(E) $f(x) = \tan^2 x$

GO ON TO THE NEXT PAGE

33. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2



34. The area of the shaded region in the figure above is represented by which of the following integrals?

(A) $\int_a^c (|f(x)| - |g(x)|)dx$

(B) $\int_b^c f(x)dx - \int_a^c g(x)dx$

(C) $\int_a^c (g(x) - f(x))dx$

(D) $\int_a^c (f(x) - g(x))dx$

(E) $\int_a^b (g(x) - f(x))dx + \int_b^c (f(x) - g(x))dx$

GO ON TO THE NEXT PAGE

35. $4\cos\left(x + \frac{\pi}{3}\right) =$

(A) $2\sqrt{3}\cos x - 2\sin x$

(B) $2\cos x - 2\sqrt{3}\sin x$

(C) $2\cos x + 2\sqrt{3}$ si

(D) $2\sqrt{3}\cos x + 2\sin x$

(E) $4\cos x + 2$

---

36. What is the average value of y for the part of the curve $y = 3x - x^2$ which is in the first quadrant?

(A) -6

(B) -2

(C) $\frac{3}{2}$

(D) $\frac{9}{4}$

(E) $\frac{9}{2}$

GO ON TO THE NEXT PAGE

37. If $f(x) = e^x \sin x$, then the number of zeros of f on the closed interval $[0, 2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

---

38. For $x > 0$, $\int \left(\frac{1}{x} \int_1^x \frac{du}{u} \right) dx =$

(A) $\frac{1}{x^3} + C$

(B) $\frac{8}{x^4} - \frac{2}{x^2} + C$

(C) $\ln(\ln x) + C$

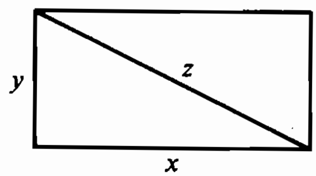
(D) $\frac{\ln(x^2)}{2} + C$

(E) $\frac{(\ln x)^2}{2} + C$

GO ON TO THE NEXT PAGE

39. If $\int_1^{10} f(x)dx = 4$ and $\int_{10}^{3} f(x)dx = 7$, then $\int_1^{3} f(x)dx =$

(A) -3 (B) 0 (C) 3 (D) 10 (E) 11



40. The sides of the rectangle above increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

(A) $\frac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5

GO ON TO THE NEXT PAGE

41. If $\lim_{x \to 3} f(x) = 7$, which of the following must be true?

I. f is continuous at $x = 3$.
II. f is differentiable at $x = 3$.
III. $f(3) = 7$

(A) None
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

---

42. The graph of which of the following equations has $y = 1$ as an asymptote?

(A) $y = \ln x$
(B) $y = \sin x$
(C) $y = \frac{x}{x+1}$
(D) $y = \frac{x^2}{x-1}$
(E) $y = e^{-x}$

GO ON TO THE NEXT PAGE

43. The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ about the x-axis is

(A) 2π (B) 4π (C) 6π (D) 9π (E) 12π

---

44. Let f and g be odd functions. If p, r, and s are nonzero functions defined as follows, which must be odd?

I. $p(x) = f(g(x))$
II. $r(x) = f(x) + g(x)$
III. $s(x) = f(x)g(x)$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

GO ON TO THE NEXT PAGE

45. The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

(A) $2\sqrt[3]{2}$ (B) $2\sqrt{2}$ (C) $2\sqrt[3]{4}$ (D) 4 (E) 8

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

1988

MATHEMATICS : CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

It may be worthwhile for you to look through the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions. The questions are printed in the booklet and on the green insert. It will be easier for you to look over all the problems on the insert; however, you should write all work for each problem in the space provided for that particular problem in the pink booklet. You may work the problems in any order. Do not spend too much time on any one problem.

Write all your answers in pencil only. Be sure to write CLEARLY and LEGIBLY. If you make an error, you may save time by crossing it out rather than trying to erase it.

Show all your work. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your final answers. All questions are given equal weight, but the parts of a particular question are not necessarily given equal weight. Credit for partial solutions will be given.

When you are told to begin, open your booklet, carefully tear out the green insert, and start work.

CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Let f be the function given by $f(x) = \sqrt{x^4 - 16x^2}$.

   (a) Find the domain of f.

   (b) Describe the symmetry, if any, of the graph of f.

Continue problem 1 on next page.

(c) Find $f'(x)$.

(d) Find the slope of the line normal to the graph of f at $x = 5$.

GO ON TO THE NEXT PAGE

2. A particle moves along the x-axis so that its velocity at any time $t \geqq 0$ is given by $v(t) = 1 - \sin(2\pi t)$.

(a) Find the acceleration $a(t)$ of the particle at any time t.

(b) Find all values of t, $0 \leqq t \leqq 2$, for which the particle is at rest.

Continue problem 2 on next page.

(c) Find the position $x(t)$ of the particle at any time t if $x(0) = 0$.

GO ON TO THE NEXT PAGE

3. Let R be the region in the first quadrant enclosed by the hyperbola $x^2 - y^2 = 9$, the x-axis, and the line $x = 5$.

(a) Find the volume of the solid generated by revolving R about the x-axis.

Continue problem 3 on next page.

(b) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the line $x = -1$.

GO ON TO THE NEXT PAGE

4. Let f be the function defined by $f(x) = 2xe^{-x}$ for all real numbers x.

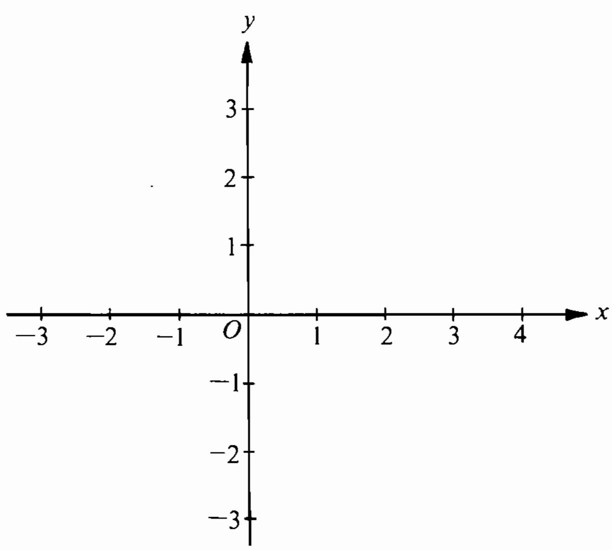
(a) Write an equation of the horizontal asymptote for the graph of f.

(b) Find the x-coordinate of each critical point of f. For each such x, determine whether $f(x)$ is a relative maximum, a relative minimum, or neither.

Continue problem 4 on next page.

(c) For what values of x is the graph of f concave down?

(d) Using the results found in parts (a), (b), and (c), sketch the graph of $y = f(x)$ in the xy-plane provided below.



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5. Let R be the region in the first quadrant under the graph of $y = \frac{x}{x^2 + 2}$ for $0 \leqq x \leqq \sqrt{6}$.

(a) Find the area of R.

(b) If the line $x = k$ divides R into two regions of equal area, what is the value of k?

Continue problem 5 on next page.

(c) What is the average value of $y = \frac{x}{x^2 + 2}$ on the interval $0 \leqq x \leqq \sqrt{6}$?

GO ON TO THE NEXT PAGE

6. Let f be a differentiable function, defined for all real numbers x, with the following properties.

(i) $f'(x) = ax^2 + bx$

(ii) $f'(1) = 6$ and $f''(1) = 18$

(iii) $\int_1^2 f(x)dx = 18$

Find $f(x)$. Show your work.

If more space is needed, continue problem 6 on next page.

END OF EXAMINATION

ANSWER KEY AND DISTRIBUTION OF RESPONSES
SECTION I
1988 AP CALCULUS AB EXAMINATION

Item No.	Correct Answer	Percentage Selecting Each Option[1] (*indicates correct answer) A	B	C	D	E	Percent Omitting Question
1.	C	6%	4%	*88%	1%	1%	1%
2.	D	21	1	1	*60	15	1
3.	A	*83	1	2	8	2	4
4.	E	3	23	1	3	*60	10
5.	A	*89	1	0	2	6	2
6.	D	2	2	4	*83	7	2
7.	D	7	1	7	*79	2	4
8.	B	10	*59	4	9	4	14
9.	E	2	3	7	1	*81	7
10.	C	2	3	*80	3	1	10
11.	A	*61	9	3	6	5	16
12.	B	4	*76	2	15	1	1
13.	A	*70	8	8	4	2	7
14.	D	5	2	7	*67	3	15
15.	B	17	*65	12	2	3	1
16.	C	11	5	*74	1	6	4
17.	D	2	5	8	*72	10	2
18.	E	3	9	6	3	*73	7
19.	B	9	*58	5	1	18	9
20.	C	15	3	*41	4	7	30
21.	C	7	3	*63	3	7	17
22.	C	3	3	*64	5	6	19
23.	B	1	*73	9	4	10	4
24.	C	4	2	*21	42	5	26
25.	B	1	*64	6	20	1	7
26.	E	5	4	7	4	*59	20
27.	E	1	19	11	5	*53	11
28.	C	18	28	*34	4	3	13
29.	B	21	*45	7	1	10	16
30.	A	*57	13	11	3	3	13

(continued on next page)

[1]For each multiple-choice question, the percentages are based on the total number of candidates presumed to have attempted the question, that is, those candidates who either marked a response to the question or left it blank but marked a response to at least one later question.

ANSWER KEY AND DISTRIBUTION OF RESPONSES (continued)

Item No.	Correct Answer	Percentage Selecting Each Option (*indicates correct answer) A	B	C	D	E	Percent Omitting Question
31.	C	7%	20%	*50%	5%	2%	16%
32.	A	*46	11	10	8	9	16
33.	A	*48	8	3	33	5	3
34.	D	15	12	2	*50	14	7
35.	B	6	*25	7	4	18	40
36.	C	1	2	*60	7	9	20
37.	D	6	7	25	*45	4	13
38.	E	1	1	13	6	*47	31
39.	E	37	1	12	2	*34	14
40.	B	9	*50	3	2	4	31
41.	A	*30	9	11	18	21	11
42.	C	5	6	*56	23	6	4
43.	B	11	*24	11	9	7	38
44.	C	19	19	*37	7	7	10
45.	D	18	18	13	*43	8	0

Advanced Placement Program®
THE COLLEGE BOARD
Master ID/Answer Sheet (MIDAS) for May 1988

SIDE 1

PLACE AP NUMBER LABEL HERE.

Use only a no. 2 pencil. Completely erase errors and stray marks.

A. SIGNATURE To maintain the security of the examination and the credibility of my AP grade, I will allow no one other than myself to see the multiple-choice questions and, when asked to do so, will seal the Section I booklet.

Write your name in the usual manner, as if signing a business letter.

B. YOUR NAME Omit spaces, hyphens, apostrophes, and Jr. or II.
Last Name - first 12 letters | First Name - first 8 letters | M.I.

C. ENTER YOUR AP NUMBER HERE

D. ADMIN. DAY OF MAY

E. TIME OF DAY 6 AM, 7 AM, 8 AM, 9 AM, 10 AM, 11 AM, 12 PM, 1 PM, 2 PM, 3 PM, 4 PM, 5 PM

F. AP EXAMINATION TO BE TAKEN OR SUBMITTED WITH THIS ANSWER SHEET

Print examination name: CALCULUS AB

Fill in the appropriate oval below for examination name and number.

07 American History	31 Computer Science A	51 French Literature	73 Music: List. & Lit.
13 Art: History of	33 Computer Science AB	55 German Language	75 Music: Theory
14 Art: Studio Drawing	36 Eng. Language & Comp.	57 Gov. & Pol.	78 Physics B
15 Art: Studio General	37 Eng. Literature & Comp.	60 Latin	80 Physics C
20 Biology	43 European History	66 ● Math. Calculus AB	87 Spanish Language
25 Chemistry	48 French Language	68 Math. Calculus BC	89 Spanish Literature

G. DO NOT COMPLETE THIS SECTION UNLESS INSTRUCTED TO DO SO.

Examination Choices

G1. Latin ① Vergil ② Cat.-Horace

G2. Physics C ① Mechanics ② Elec. & Mag.

G3. Gov. & Pol. ① American ② Comparative

G4. Essay Choices
Fill in the ovals under the numbers of the essay questions you answered in this examination.
1 2 3 4 5 6 7 8 9 10

G5. If this answer sheet is for the French, German, or Spanish Language Examination, please answer the following questions. (Your responses will not affect your grade.)

1. Have you lived or studied for one month or more in a country where the language of the exam you are now taking is spoken? Yes No
2. Do you regularly speak or hear the language at home? Yes No

SCHOOL USE ONLY

Fee Reduction Granted?

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE.
IF A QUESTION HAS ONLY FOUR ANSWER OPTIONS, DO NOT MARK THE OPTION E.

Question	Marked	Question	Marked
1	C	31	C
2	D	32	A
3	A	33	A
4	E	34	D
5	A	35	B
6	D	36	C
7	D	37	D
8	B	38	E
9	E	39	E
10	C	40	B
11	A	41	A
12	B	42	C
13	A	43	B
14	D	44	C
15	B	45	D
16	C		
17	D		
18	E		
19	B		
20	C		
21	C		
22	C		
23	B		
24	C		
25	B		
26	E		
27	E		
28	C		
29	B		
30	A		

Questions 46–150: no marks.

ETS USE ONLY

	R	W	FS
PT1			
PT2			
PT3			
PT4			
TOT			
EQ			
TA1			
TA2			

ETS USE ONLY

180TF17P1065
Q1622-06
I.N.239993

SOLUTIONS AND SCORING GUIDES
SECTION II
1988 AP CALCULUS AB EXAMINATION

The solutions to the free-response questions that follow are intended to illustrate the general level of detail expected of candidates, and the scoring standards indicate how points were awarded to the main aspects of the answers. However, approaches to a problem may vary and, even with the same approach, the written solutions may vary in the way that the different steps are presented. Within the framework of points indicated, the Readers exercise latitude in interpreting the correctness of the solutions.

For example, credit is sometimes awarded for steps not written down, if it is clear that the candidate must have taken those steps to arrive at the result indicated. However, candidates are advised to show their work in order to minimize the risk of not receiving credit for it. Further, if one part of a problem depends on a value obtained in a previous part, and that value is incorrect, full credit is awarded for the latter part if it is done correctly, even though an incorrect result may have been obtained.

Readers often worked with scoring guides more detailed than those presented here in order to award points for correct approaches or to subtract points for calculus or other mathematical errors. In addition, Readers sometimes had scoring guides available for common incorrect approaches that nevertheless revealed some understanding of the calculus involved. In this way, a high degree of consistency in grading is obtained.

1988 Calculus AB

1. Let f be the function given by $f(x) = \sqrt{x^4 - 16x^2}$.

   (a) Find the domain of f.

   (b) Describe the symmetry, if any, of the graph of f.

   (c) Find $f'(x)$.

   (d) Find the slope of the line <u>normal</u> to the graph of f at $x = 5$.

Solution	Distribution of Points
(a) $x^4 - 16x^2 \geq 0$ $x^2(x^2 - 16) \geq 0$ $x^2 \geq 16$ or $x = 0$ $\lvert x\rvert \geq 4$ or $x = 0$	(a) 3: { 1: for radicand ≥ 0 1: for $\lvert x\rvert \geq 4$ 1: for $x = 0$
(b) Symmetric about the y-axis	(b) 1: for correct answer
(c) $f'(x) = \frac{1}{2}(x^4 - 16x^2)^{-\frac{1}{2}}(4x^3 - 32x)$ $= \dfrac{2x(x^2-8)}{\lvert x\rvert\sqrt{x^2-16}}$	(c) 3: for correct derivative
(d) $f'(5) = \dfrac{2(5)(25-8)}{5\sqrt{25-16}}$ $= \dfrac{10(17)}{5\sqrt{9}} = \dfrac{170}{15} = \dfrac{34}{3}$ $\therefore$ slope of normal line is $-\dfrac{3}{34}$	(d) 2: { 1: for evaluating $f'(x)$ found in (c) at $x = 5$ 1: for slope of normal

2. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 1 - \sin(2\pi t)$.

(a) Find the acceleration $a(t)$ of the particle at any time t.

(b) Find all values of t, $0 \leq t \leq 2$, for which the particle is at rest.

(c) Find the position $x(t)$ of the particle at any time t if $x(0) = 0$.

Solution	Distribution of Points
(a) $a(t) = v'(t)$ $= -2\pi \cos(2\pi t)$	(a) 2: for correct differentiation of velocity
(b) $v(t) = 0$ $1 - \sin(2\pi t) = 0$ or $1 = \sin(2\pi t)$ $2\pi t = \frac{\pi}{2} + 2k\pi$, where $k = 0, \pm 1, \pm 2, \ldots$ and $0 \leq t \leq 2$ $\therefore\ t = \frac{1}{4}$ and $t = \frac{5}{4}$	(b) 3: { 1: for $1 - \sin(2\pi t) = 0$ 1: for $t = \frac{1}{4}$ 1: for $t = \frac{5}{4}$ }
(c) $x(t) = \int v(t)\,dt$ $= \int [1 - \sin(2\pi t)]\,dt$ $= t + \frac{1}{2\pi}\cos(2\pi t) + C$ $x(0) = 0 = 0 + \frac{\cos(0)}{2\pi} + C$ $\therefore\ x(t) = t + \frac{1}{2\pi}\cos(2\pi t) - \frac{1}{2\pi}$	(c) 4: { 2: for correct antiderivative of $v(t)$ 1: for $x(0) = 0$ 1: for finding value of C }

3. Let R be the region in the first quadrant enclosed by the hyperbola $x^2 - y^2 = 9$, the x-axis, and the line $x = 5$.

(a) Find the volume of the solid generated by revolving R about the x-axis.

(b) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the line $x = -1$.

Solution	Distribution of Points
(a)	(a)

Solution

(a) Discs:

$$V = \pi \int_3^5 (x^2-9)\,dx$$

$$= \pi\left[\frac{1}{3}x^3 - 9x\right]_3^5$$

$$= \pi\left[\left(\frac{125}{3} - 45\right) - (9 - 27)\right] = \frac{44}{3}\pi$$

or

Shells:

$$V = 2\pi \int_0^4 \left(5 - \sqrt{9+y^2}\right) y\,dy$$

$$= 2\pi\left[\frac{5}{2}y^2 - \frac{1}{3}(9+y^2)^{\frac{3}{2}}\right]_0^4$$

$$= 2\pi\left(40 - \frac{125}{3} + \frac{27}{3}\right) = \frac{44}{3}\pi$$

Distribution of Points

(a) 5:
- 2: for a correct integrand
- 1: for appropriate limits and $k\pi$
- 1: for correct antiderivative
- 1: for substitution and/or evaluation

Solution

(b) Shells:

$$V = 2\pi\int_3^5 (x+1)y\,dx$$

$$= 2\pi\int_3^5 (x+1)\sqrt{x^2-9}\,dx$$

or

Washers:

$$V = \pi\int_0^4 [36 - (x+1)^2]\,dy$$

$$= \pi\int_0^4 \left[36 - \left(\sqrt{9+y^2} + 1\right)^2\right]dy$$

Distribution of Points

(b) 4:
- 3: for a correct integrand
- 1: for appropriate limits and $k\pi$

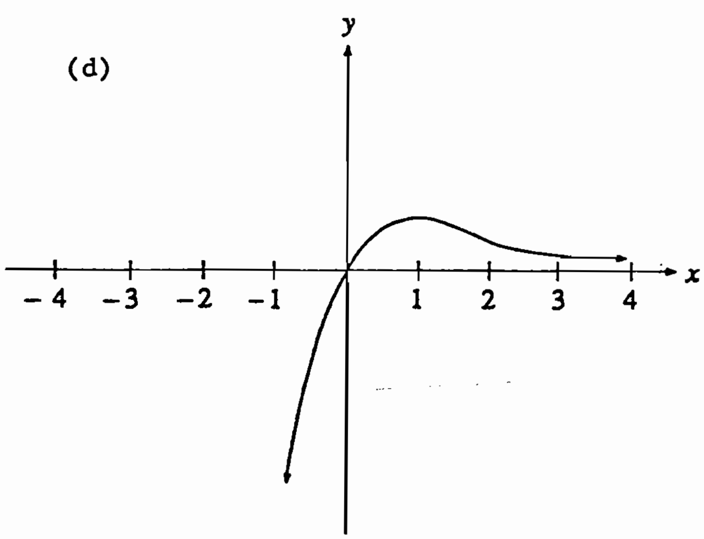
4. Let f be the function defined by $f(x) = 2xe^{-x}$ for all real numbers x.

(a) Write an equation of the horizontal asymptote for the graph of f.

(b) Find the x-coordinate of each critical point of f. For each such x, determine whether $f(x)$ is a relative maximum, a relative minimum, or neither.

(c) For what values of x is the graph of f concave down?

(d) Using the results found in parts (a), (b), and (c), sketch the graph of $y = f(x)$ in the xy-plane provided below.

Solution	Distribution of Points
(a) $y = 0$	(a) 1: for correct equation
(b) $f'(x) = 2(-xe^{-x} + e^{-x})$ $= 2e^{-x}(1 - x)$ critical point at $x = 1$ relative maximum at $x = 1$	(b) 3: { 1: for correct derivative 1: for critical value for f' 1: for identifying critical point as relative maximum
(c) $f''(x) = 2e^{-x}(-1) + (-2e^{-x})(1-x)$ $= 2e^{-x}(x - 2)$ Concave down when $2e^{-x}(x - 2) < 0$ $(x - 2) < 0$ $x < 2$	(c) 2: { 1: for correct $f''(x)$ for $f'(x)$ found in (b) 1: for correct interval
(d) 	(d) 3: for graph consistent with information found in (a), (b), and (c)

5. Let R be the region in the first quadrant under the graph of $y = \frac{x}{x^2 + 2}$ for $0 \leqq x \leqq \sqrt{6}$.

(a) Find the area of R.

(b) If the line $x = k$ divides R into two regions of equal area, what is the value of k?

(c) What is the average value of $y = \frac{x}{x^2 + 2}$ on the interval $0 \leqq x \leqq \sqrt{6}$?

Solution	Distribution of Points
(a) $A = \int_0^{\sqrt{6}} \frac{x}{x^2 + 2}\,dx$ $= \frac{1}{2}\ln(x^2 + 2)\Big\|_0^{\sqrt{6}}$ $= \frac{1}{2}\ln 8 - \frac{1}{2}\ln 2 = \ln 2$	(a) 3: { 1: for correct integral 1: for antiderivative 1: for evaluation
(b) $\frac{1}{2}\ln 2 = \int_0^{k} \frac{x}{x^2 + 2}\,dx$ $= \frac{1}{2}\ln(x^2 + 2)\Big\|_0^{k}$ $= \frac{1}{2}\ln(k^2 + 2) - \frac{1}{2}\ln 2$ $\therefore \frac{1}{2}\ln(k^2 + 2) = \frac{1}{2}\ln 2 + \frac{1}{2}\ln 2$ or $\ln(k^2 + 2) = \ln 4$ $\therefore k^2 + 2 = 4$ and $k = \sqrt{2}$	(b) 4: { 1: for a correct equation involving integral(s) 1: for antiderivative 1: for evaluation of integral(s) 1: for finding the value of k
(c) Average value $= \frac{1}{\sqrt{6} - 0}\int_0^{\sqrt{6}} \frac{x}{x^2 + 2}\,dx$ $= \frac{1}{\sqrt{6}}\ln 2$	(c) 2: { 1: for correct integral 1: for evaluation

6. Let f be a differentiable function, defined for all real numbers x, with the following properties.

(i) $f'(x) = ax^2 + bx$

(ii) $f'(1) = 6$ and $f''(1) = 18$

(iii) $\int_1^2 f(x)\,dx = 18$

Find $f(x)$. Show your work.

Solution	Distribution of Points
$a + b = 6$ $f''(x) = 2ax + b$ $2a + b = 18$ $\therefore a = 12,\ b = -6$ and $f'(x) = 12x^2 - 6x$	4: 1: for $a + b = 6$ 1: for $f''(x)$ 1: for $2a + b = 18$ 1: for $a = 12,\ b = -6$
$f(x) = \int (12x^2 - 6x)\,dx$ $= 4x^3 - 3x^2 + C$ $\int_1^2 (4x^3 - 3x^2 + C)\,dx = x^4 - x^3 + Cx \Big\|_1^2$ $= (16 - 8 + 2C) - (1 - 1 + C)$ $= 8 + C = 18$ or $C = 10$ $\therefore f(x) = 4x^3 - 3x^2 + 10$	5: 1: for an antiderivative of f' 1: for including C 1: for antiderivative of f 1: for evaluating integral and equating it to 18 1: for finding value of C

HOW THE AP GRADES IN CALCULUS AB ARE DETERMINED

Grades for the Advanced Placement Examinations are reported on a five-point scale, ranging from 1 to 5. Before these grades are determined, a number of intermediate scoring steps take place. First, the answer sheet for the multiple-choice section is machine-scored. This score is the number of correct answers minus a fraction of the number of incorrect answers; negative scores are set equal to zero. Second, scores are assigned to individual problem sets in the free-response section by Readers at the AP Reading. These scores are based on detailed scoring standards established by the Chief Reader and the Readers. Third, the scores on the free-response questions and the multiple-choice section are weighted according to formulas determined in advance by the AP Mathematics Development Committee to yield a single composite score for each candidate. In Calculus AB, the six problems that constitute the free-response section are weighted equally to form the total free-response score. The total free-response score and the total multiple-choice score are then weighted equally in forming the composite score, for which the maximum value was 108 in 1988. Finally, the conversion from the composite scores to the reported grades is determined by setting four cut-points on the composite score scale which are used to determine the ranges of composite scores that make up the five possible grades. The setting of these cut-points for each examination is based on the judgment of the Chief Reader in consultation with ETS professional staff.

A variety of information is available to assist the Chief Reader in judging which papers have scored high enough to receive each of the grades. Computer printouts with complete distributions of scores on each portion of the multiple-choice and free-response sections of the examination are provided along with totals for each section and the composite score total. With these figures and special statistical tables presenting score distributions from previous years, the Chief Reader can calibrate the examination against the results of other years and can evaluate the section-by-section performance on the current examination. Assessments are also made of the examination itself as well as the reliability of the grading. Finally, computer rosters containing the complete breakdown of scores for thousands of candidates enable the Chief Reader to analyze patterns of performance. On the basis of professional judgment regarding the quality of performance represented by the achieved scores, the Chief Reader determines the candidates' final AP grades.

The grade distributions for the 1988 AP Mathematics Examination: Calculus AB are shown below, with the percentage indicated at each grade level, together with the mean and standard deviation.

	Examination Grade	Number of Students	Percent at Grade
Extremely well qualified	5	9,666	17.8
Well qualified	4	12,295	22.7
Qualified	3	15,003	27.7
Possibly qualified	2	8,547	15.8
No recommendation	1	8,724	16.1
Total Number of Students*		54,235	
Mean Grade		3.10	
Standard Deviation		1.31	

*This total differs from the total number of candidates given on pages 60 and 61 because the statistical summaries were produced at different times and hence different candidate data were available.

RELIABILITY OF CLASSIFICATION
1988 AP CALCULUS AB EXAMINATION

The classification reliability of AP grades can be examined by using a recently developed statistical technique that makes it possible to estimate the consistency and accuracy of decisions based on those grades. The consistency of the decisions is the extent to which they would agree with the decisions that would have been made if the candidate had taken a different form of the AP Calculus AB Exam, equal in difficulty and covering the same content as the form the candidate actually took. The accuracy of the decisions is the extent to which they would agree with the decisions that would be made if each candidate could somehow be tested with all possible forms of the exam.

The table below shows the decision consistency and accuracy of the 1988 AP Calculus AB Examination. Each number in the table indicates the estimated percentage of candidates who would be consistently classified as above or below the 2 to 3 and the 3 to 4 grade boundaries.

Estimated Consistency and Accuracy of Decisions Based on AP Grades for the 1988 AP Calculus AB Examination

Estimated Percentage of Candidates Who Would Be Reclassified the Same Way on the Basis of:			
Another Form		Average of All Forms	
2-3 boundary	3-4 boundary	2-3 boundary	3-4 boundary
93%	92%	95%	94%

The percentages in the table are estimates--candidates never actually took more than one form of the exam--and are based on data from a representative sample of the total group of candidates who took the 1988 AP Calculus AB Exam. Research results indicate that these estimates are biased in an upward direction and, therefore, overestimate the actual consistency and accuracy of decisions based on AP grades.

SCORING WORKSHEET
1988 AP CALCULUS AB EXAMINATION

SECTION I: MULTIPLE-CHOICE (TOTAL):

_______ - (1/4 x _______) = ______________________________

Number correct

Number wrong

Multiple-choice score
(Round to nearest whole number. If less than zero, enter zero.)

SECTION II: FREE-RESPONSE:

Scores for individual questions

Question 1 __________ (Out of 9)

2 __________ (Out of 9)

3 __________ (Out of 9)

4 __________ (Out of 9)

5 __________ (Out of 9)

6 __________ (Out of 9)

Sum = ____________________
Free-response score

COMPOSITE SCORE:

1.200 x ________________ = ____________________

Multiple-choice score

Weighted multiple-choice score

____________________ + _______________ = ____________________

Weighted multiple-choice score

Free-response score

Composite score

AP GRADE:

Composite Score Range*	AP Grade
83 - 108	5
68 - 82	4
48 - 67	3
32 - 47	2
0 - 31	1

*This composite score range is for the 1988 examination only.

DISTRIBUTION OF SCORES
SECTION II
1988 AP CALCULUS AB EXAMINATION

	Free-Response Questions					
Score	1 (9*)	2 (9*)	3 (9*)	4 (9*)	5 (9*)	6 (9*)
9	3,364	8,926	6,783	7,048	11,050	21,138
8	9,108	9,725	3,421	3,878	5,721	5,596
7	11,382	7,406	4,643	6,502	4,101	2,333
6	10,626	5,900	5,894	5,320	2,704	4,156
5	7,938	4,447	7,904	5,713	7,411	6,997
4	5,421	3,946	5,146	5,437	3,611	3,407
3	3,323	3,351	4,189	4,756	5,539	2,456
2	1,384	3,508	3,707	4,679	4,565	2,387
1	1,016	2,514	3,567	3,677	4,219	2,235
0	928	4,767	9,236	7,480	5,569	3,785
Total Number of Candidates**	54,490	54,490	54,490	54,490	54,490	54,490
Mean	5.90	5.56	4.40	4.58	4.98	6.23
Standard Deviation	2.00	2.90	2.99	2.97	3.09	2.98
Mean as % of Max. Possible Score	65.56	61.78	48.89	50.89	55.33	69.22

*Maximum possible score

**This total differs from the total number of candidates given on pages 57 and 61 because the statistical summaries were produced at different times and hence different candidate data were available.

MULTIPLE-CHOICE SCORES AND AP GRADES
1988 AP CALCULUS AB EXAMINATION

Multiple-Choice Score	AP Grade 1	2	3	4	5	Total	
32 to 45	0 0.0%	0 0.0%	56 0.5%	1,975 18.9%	8,411 80.5%	10,442 100.0%	(19.1%)
26 to 31	0 0.0%	28 0.2%	2,380 20.3%	8,030 68.4%	1,294 11.0%	11,732 100.0%	(21.5%)
18 to 25	52 0.3%	2,278 14.6%	10,926 70.0%	2,344 15.0%	4 0.0%	15,604 100.0%	(28.6%)
12 to 17	1,514 18.1%	5,162 61.7%	1,685 20.1%	3 0.0%	0 0.0%	8,364 100.0%	(15.3%)
0 to 11	7,252 86.2%	1,132 13.5%	31 0.4%	0 0.0%	0 0.0%	8,415 100.0%	(15.4%)
Total	8,818 16.2%	8,600 15.8%	15,078 27.6%	12,352 22.6%	9,709 17.8%	54,557* 100.0%	(100.0%)

This table shows the statistical relationship between candidates' AP grades on the 1988 AP Calculus AB Examination and their scores on the multiple-choice portion of the examination. The multiple-choice scores have been divided into five categories, corresponding to the five AP grade levels. Each multiple-choice score category contains approximately the same percentage of the scores as the corresponding AP grade level. The table shows the number and the percentage of the students in each multiple-choice score category who received each AP grade. For example, there were 10,442 students with multiple-choice scores of 32 to 45, and 8,411 of these students, or 80.5 percent, received an AP grade of 5. The percentages shown in parentheses at the far right of the table indicate the percentage of all the candidates who had multiple-choice scores in that category. For example, 19.1 percent of all the candidates had multiple-choice scores of 32 to 45.

Of the candidates with multiple-choice scores of 32 or higher (corrected for guessing), all but 0.5 percent earned AP grades of 4 or 5, and the majority earned a 5. Of those with multiple-choice scores of 26 to 31, the majority earned a 4, while most others earned a 3 or a 5. Of those with multiple-choice scores of 18 to 25, the majority earned a 3, while most others earned a 2 or a 4. Of those with multiple-choice scores of 12 to 17, most earned a 2, while most others earned a 1 or a 3. Of the candidates with multiple-choice scores of 11 or less, the majority earned a 1, while most others earned a 2.

*This total includes only those candidates who requested that their AP grades be reported. It also differs from the total number of candidates given on pages 57 and 60 because the statistical summaries were produced at different times and hence different candidate data were available.

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