AP Calculus: Midterm Review 2016 NAME ____________________

Limits: $\lim\limits_{x \to a} f(x) = L$

EX 1] Find $\lim\limits_{x \to -2} \dfrac{x+2}{x^2 - x - 6}$.

EX 2] Find $\lim\limits_{x \to 3} \dfrac{x+2}{x^2 - x - 6}$.

EX 3] Find $\lim\limits_{x \to 0} \dfrac{\sin(x)}{x}$.

EX 4] Find $\lim\limits_{x \to 0} \dfrac{\sin(4x)}{x}$.

EX 5] Find $\lim\limits_{x \to 0} \dfrac{\sin(x)}{3x}$.

EX 6] Find $\lim\limits_{x \to 0} \dfrac{1 - \cos(x)}{x}$.

EX 7] Find $\lim\limits_{x \to \infty} \dfrac{x+2}{x^2 - x - 6}$.

EX 8] Find $\lim\limits_{x \to \infty} \sin(x)$.

Continuity

EX 1] Determine any removable discontinuity of $f(x) = \dfrac{x^2 + x - 6}{x^2 + 5x + 6}$.

EX 2] Let $f(x) = \begin{cases} x^2, & x \geq 2 \\ 4 - 2x, & x < 2 \end{cases}$. Determine if *f(x)* is continuous at $x = 2$.

Continuity and Differentiability

EX 1] If $f(x)$ is continuous at $x = a$, which of the following is **NOT** necessarily true? Why?

a) $f(a)$ is defined.

b) $f'(a)$ exists.

c) $\lim\limits_{x \to a} f(x)$ exists.

d) $\lim\limits_{x \to a} f(x) = f(a)$.

e) $\lim\limits_{x \to a-} f(x) = \lim\limits_{x \to a+} f(x)$

EX 2] Find an equation of the tangent line to the graph $f(x) = x^3 - 2$ at the point (2,6).

Differentiation

EX 1] If $\lim_{x \to a} f(x) = 10$, what must be true?

a) $f'(a)$ exists

b) $f(x)$ is continuous at $x = a$.

c) $f(x)$ is defined at $x = a$.

d) $f(a) = 10$

e) none of these

EX 2] If $y = 2\sin\left(\frac{x}{2}\right)$, find $\frac{d^2y}{dx^2}$.

EX 3] Which of the following is true for the function $f(x) = \begin{cases} x^2, & x < 2 \\ 4(x-1), & x \geq 2 \end{cases}$ at $x = 2$?

a) Undefined
b) Continuous, but **<u>NOT</u>** differentiable
c) Differentiable, but **<u>NOT</u>** continuous
d) Neither continuous nor differentiable
e) Both continuous and differentiable

EX 4] Let $g(x) = x^{\frac{1}{3}} - \frac{64}{x}$. Find $g'(8)$.

Chain Rule

EX 1] Find y' if $y = \sin(3x)$.

EX 2] Find $\frac{dy}{dx}$ if $y = (4x - 3)^7$.

The Product Rule

EX 3] If $g(x) = x\sin(x^2)$, find $g'(x)$.

EX 4] Let $f(x) = (x-1)^2 \sin(x)$. Find the slope of the tangent to $f(x)$ at $x = 0$.

The Quotient Rule

EX 5] Find the derivative of $y = \frac{2x + 1}{x - 2}$.

Implicit Differentiation

EX 7] Find $\frac{dy}{dx}$ if $x^2y + \sin(y) = 2$.

EX 8] Find $\frac{dy}{dx}$ if $\cot(xy) = x$.

Intermediate Value Theorem

EX 1] Show that $f(x) = x^3 + 2x - 1$ has a root in $[0,1]$.

Mean Value Theorem

EX 2] If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b), the Mean Value Theorem guarantees a point $x = c$ between $x = a$ and $x = b$ such that…

a) $f'(c) = \dfrac{f(a) - f(b)}{b - a}$

b) $c = \dfrac{b - a}{f(b) - f(a)}$

c) $f'(c) = \dfrac{b - a}{f(b) - f(a)}$

d) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

e) none of these

Rolle's Theorem

EX 3] Let $f(x) = x^3 - 3x^2$ on the interval $[0,3]$. If Rolle's Theorem applies, find the value of c, where $0 \le c \le 3$, which is guaranteed by Rolle's Theorem.

EX 4] Draw a graph that demonstrates each theorem: IVT, MVT, & Rolle's

Differentials

EX 1] Demand Function: A company finds that the deman for its commodity is $p = 75 - \frac{1}{4}x$, if x changes from 7-8, find and compare the values of Δp and dp.

Position verses Time

EX 1] If you take at trip of 100 miles and it takes two hours to make the trip, your average velocity is 50 mph during the entire trip. What does the Mean Value Theorem guarantee happened on your trip?

EX 2] Let $f(x) = x^2 - 2x + 5$. Find the value of $x = c$ that satisfies the Mean Value Theorem on the interval [1,3].

Velocity and Acceleration

EX 1] Suppose a rock is dropped off a cliff and its height is given by $s(t) = -16t^2 + 960$.

a) Find the initial (t = 0) position (ft) and velocity $\left(\frac{\text{ft}}{\text{sec}}\right)$.

b) Find the average velocity $\left(\frac{\text{ft}}{\text{sec}}\right)$ over the interval [0,4].

c) At what time (sec) does the rock hit the ground?

d) What is the velocity $\left(\frac{\text{ft}}{\text{sec}}\right)$ when the rock hits the ground?

Related Rates

EX 1] An aluminum can full of a liquid has sprung a leak of 10 $\frac{\text{in}^3}{\text{min}}$.
The can has a height of 12 in and a radius of 4 in.
What is the rate at which the height of the liquid is falling?

EX 2] Water is dripping out of an inverted cone at the rate of $2\frac{\text{ft}^3}{\text{min}}$. The cone has an altitude of 10 ft and a base with a diameter of 6 ft. When the cone is ***half*** full, determine how fast the surface of the water is falling.

First Derivative

EX 1] Find the intervals where the function is increasing and decreasing and relative extrema for $f(x) = x^4 - \frac{16}{3}x^3 - 10x^2 + 2$.

Extrema

EX 1] Determine the absolute extrema of $f(x) = x^2\left(x^2 - 8\right)$ on [-1,3].

Concavity and Point of Inflection

EX 1] Find the intervals of concavity and the points of inflection for $f(x) = \frac{1}{3}x^4 - 8x^2 + 8$.

Optimization

EX 1] Determine the maximum area of a rectangle that can be inscribed in the parabola $y = 9 - x^2$.

Asymptotes

Vertical Asymptotes:

EX 1] Find the vertical asymptote of $f(x) = \dfrac{x + 2}{x^2 - x - 6}$.

EX 2] Let $f(x) = \dfrac{x - 2}{x^2 - 4}$. Why is the line $x = -2$ an asymptote?

Horizontal Asymptotes:

EX 3] Find the horizontal asymptote of $f(x) = \dfrac{x^2 + 2x}{x^2 - 1}$.

Graphs from Graphs

EX 1]

Sketch a graph of the function from the given information.

f is continuous.
$f(1) = 3, \quad f(4) = 2, \quad f(8) = 3, \quad f(6) = 1$
y-intercept $= 4$
$f'(1) = f'(6) = 0$
$f'(x) > 0$ for $(6, \infty)$
$f'(x) < 0$ for $(-\infty, 1) \cup (1, 6)$
$f''(1) = f''(4) = 0$
$f''(x) < 0$ for $(1, 4)$
$f''(x) > 0$ for $(-\infty, 1) \cup (4, \infty)$